Chapter 12
Analysis of Variance (ANOVA)

Overview

- Recall: 2-sample t-test
  - H_0: $\mu_1 = \mu_2$
  - H_A: $\mu_1 \neq \mu_2$

Overview (cont.)

- However, what do we do if there are more than 2 groups to compare?
  - We could conduct all possible pairwise comparisons.
    - This is inefficient (particularly if the number of groups is large)
    - Multiple testing issue→inflated Type I error

ANOVA

- ANOVA is a methodology that provides a opportunity to compare the means of 2 or more groups.
  - It is more efficient than all possible pairwise comparisons
  - It controls for the multiple testing issue.
  - Note: when the number of groups is 2, then ANOVA simplifies to the 2-sample t-test.
  - Similar to the t-test, ANOVA is used when the outcome variable of interest is continuous.

ANOVA

- Hypotheses:
  - H_0: Population means of ALL groups are equal
  - H_A: At least one population mean is not equal to the others.

ANOVA

- ANOVA attempts to answer questions about group means, however it does so by "analysis of variance".
- The key idea is that total variability may be divided into: (1) variability within groups, and (2) variability between groups.
  - If there is no difference between group means then these 2 sources of variability should be similar.
    - Thus by comparing these sources of variability, we can compare the group means.

ANOVA

- Has several assumptions that must be checked.
- Utilizes an F-test with a numerator and denominator degrees of freedom
- We obtain an ANOVA Table which summarizes the sources of variation, degrees of freedom, and F-test results (p-value, etc.)

ANOVA

- If the null hypothesis (that all of the group means are equal) is not rejected, then in general there is no need to investigate further.
  - One may still investigate power and potential Type II error.
- However, if the null hypothesis is rejected, then sufficient evidence has been found to conclude that at least one of the means (and possibly more) is different from the others.

ANOVA

- Where are the differences?
- Which means are different?
- We answer these questions using *post-hoc* tests.

ANOVA *post-hoc* tests

- Overall experiment-wise error rate can be controlled using multiple comparison procedures, e.g.
  - Bonferroni
  - Tukey
  - Scheffe
  - Others

Nonparametric Analog

- If the normality assumptions do not hold then nonparametric methods can be applied.
  - Kruskal-Wallis one-way ANOVA
- If there is more than one factor (for example a blocking factor), then one may elect a two-way (or higher level) ANOVA. One may also look at interactions between factors.
  - Friedman two-way ANOVA

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