Confidence Intervals

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Statistical Inference

- Statistical Inference
  - Inferences regarding a population are made based on a sample
    - Inferences about population parameters (e.g., 💻) are made by examining sample statistics (e.g., sample mean)

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Statistical Inference

- Statistical Inference
  - 2 primary approaches
    - Hypothesis Testing (of a population parameter)
    - Estimation (of a population parameter)
      - Point Estimation
        - » Sample mean is an estimate of the population mean
      - Interval Estimation
        - » Confidence Intervals
- Hypothesis Testing vs. Estimation
  - Closely related

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Confidence Interval

- A range of values associated with a parameter of interest (such as a population mean or a treatment effect) that is calculated using the data, and will cover the ***TRUE*** parameter with a specified probability (if the study was repeated a large number of times)

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Confidence Interval (CI)

- Based on sample statistics (estimates of population parameters)
- The width of the CI provides some information regarding the precision of the estimate
- Provides both a range of plausible values and a test for the parameter of interest

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Confidence Interval

- Roughly speaking: an interval within which we expect the true parameter to be contained.
- Based on the notion of repeated sampling (similar to hypothesis testing)

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Confidence Interval

- The "percent" indicates the probability (based on repeated sampling) that the CI covers the TRUE parameter
  - Not the probability that the parameter falls in the interval
    - The CI is the random entity (that depends on the random sample)
    - The parameter is fixed
    - A different sample would produce a different interval, however the parameter of interest remains unchanged.

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Illustration

- We are analyzing a study to determine if a new drug decreases LDL cholesterol.
  - We measure the LDL of 100 people before administering the drug and then again after 12 weeks of treatment.
  - We then calculate the mean change (post-pre) and examine it to see if improvement is observed.

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Illustration

- Assume now that the drug has no effect (i.e., the true mean change is 0). Note that in reality, we never know what the true mean is.
- We perform the study and note the mean change and calculate a 95% CI
- Hypothetically, we repeat the study an infinite number of times (each time re-sampling 100 new people)

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Illustration

- Study 1, Mean=–7, 95% CI (-12, 2)
- Study 2, Mean=–2, 95% CI (-9, 5)
- Study 3, Mean=4, 95% CI (-3, 11)
- Study 4, Mean=0, 95% CI (-7, 7)
- Study 5, Mean=-5, 95% CI (-12, 2)
- .
- .
- .

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Illustration

- Remember the true change is zero (i.e., the drug is worthless)
- 95% of these intervals will cover the true change (0).
  - 5% of the intervals will not cover 0
- In practice, we only perform the study once.
- We have no way of knowing if the interval that we calculated is one of the 95% (that covers the true parameter) or one of the 5% that does not.

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Example

- Two treatments are being compared with respect to "clinical response" for the treatment of nosocomial (hospital-acquired) pneumonia
- The 95% CI for the difference in response rates for the two treatment groups is (-0.116, 0.151)
- What does this mean?

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Example (continued)

- Based upon the notion of repeated sampling, 95% of the CIs calculated in this manner would cover the true difference in response rates.
  - We do not know if we have one of the 95% that covers the true difference or one of the 5% that does not.
- Thus we are 95% confident that the true between-group difference in the proportion of subjects with clinical response is between -0.116 and 0.151

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Caution

- Thus, every time that we calculate a 95% CI, then there is a 5% chance that the CI does not cover the quantity that you are estimating.
  - If you perform a large analyses, calculating many CIs for many parameters, then you can expect that 5% of the will not cover the the parameters of interest.

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CIs and Hypothesis Testing

- To use CIs for hypothesis testing: values between the limits are values for which the null hypothesis would not be rejected
  - At the α=0.05 level:
    - We would fail to reject H_0: treatment change=0 since 0 is contained in (-0.116, 0.151)
    - We would reject H_0: treatment change=-20 since -20 is not contained in (-0.116, 0.151)

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CIs and Hypothesis Testing

- What would be the conclusion of these hypothesis tests if we wanted to test at α=0.01 or α=0.10?

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Confidence Interval Width

- It is desirable to have narrow CIs
  - Implies more precision in your estimate
  - Wide CIs have little meaning
- In general, with all other things being equal:
  - Smaller sample size → wider CIs
  - Higher confidence → wider CIs
  - Larger variability → wider CIs

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In Practice

- It is a good idea to provide confidence intervals in an analyses
  - They provide both a test and an estimate of the magnitude of the effect (which p-values do not provide)

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For Illustration

- Consider the neuropathy example (insert neuropathy_example.pdf)
- Sensitivity 48.5
  - w/ 95% CI (36.9, 60.3)

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CIs

- Similar to hypothesis testing:
  - We may choose the confidence level (e.g., 95% → α=0.05)
  - CIs may be:
    - 1-sided: (-∞, value) or (value, ∞), or
    - 2-sided (value, value)

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CIs

- Some things to think about
  - Scale of the variable(s): continuous vs. binary (next class)
  - 1-sample vs. 2-sample
    - CI for a population mean
    - CI for the difference between 2 population means
  - ☞ known or unknown

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CIs

- Insert CI1.pdf

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